Valid Inequalities and a Branch-and-Cut Algorithm for multi-depot vehicle routing problems

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Abstract

We present a generic branch-and-cut framework for solving routing problems with multiple depots and asymmetric cost-structures, which consist in finding a set of cost minimizing (capacitated) vehicle tours in order to fulfill a set of customer demands. The backbone of the branch-and-cut framework is a series of valid inequalities, and accompanying separation algorithms, exploiting the asymmetric cost-structure in directed graphs. We derive three new classes of so-called DK inequalities that can eliminate subtours, enforce tours to be linked to a single depot, and impose bounds on the number of allowed customers in a tour. In addition, other well-known valid inequalities for solving vehicle routing problems are generalized and adapted to be valid for routing problems with multiple depots and asymmetric cost-structures. The resulting branch-and-cut framework is tested on four specific problem variants, for which we develop a new set of large-scale benchmark instances. The new DK inequalities are able to reduce root node optimality gaps by up to 67.2% compared to existing approaches in the literature. The overall branch-and-cut framework is effective as, e.g., Asymmetric Multi-Depot Traveling Salesman Problem instances containing up to 400 customers and 50 depots can be solved to optimality, for which only solutions of instances up to 300 customer nodes and 60 depots were reported in the literature before.

Keywords: Branch, and, Cut, Valid Inequalities, Asymmetric, Multi, Depot, Vehicle Routing

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