Multi-Trip Vehicle Routing Problems: Variants, Formulations, and Exact Methods
Tutorial - VeRoLog 2019, Seville

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Introduction

• Most of the literature on the Vehicle Routing Problem (VRP) addresses problems where each vehicle can perform at most one trip per day
• Many contributions on VRPs where vehicles can perform multiple trips have been published in the last decade
• These problems are called Multi-Trip Vehicle Routing Problems (MTVRP)
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• These problems are called Multi-Trip Vehicle Routing Problems (MTVRP)
Motivation

- Such an increasing interest in MTVRPs is due, e.g., to new practices in city logistics and last-mile delivery
- The need of limiting noise and pollution in city centers requires the usage of small vans, electric vehicles, and/or drones and forbids large trucks from entering city centers
- The limited capacity/autonomy of these vehicles force them to perform multiple trips and to return to the depot to reload multiple times over the day
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Main Question of this Tutorial

What is the best model to solve an MTVRP to optimality?

Based on the state-of-the-art exact methods for lots of VRPs...

Set Partitioning Models!
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## Definition of the Multi-Trip VRP I

### Input Data

- **N** set of customers
- **V** vertex set, $V = N \cup \{0\}$, where 0 is the depot
- **A** arc set, $A = \{(i,j) \mid i, j \in V : i \neq j\}$
- **G** directed graph, $G = (V, A)$
- **$t_{ij}$** travel time of arc $(i,j) \in A$
- **K** fleet of identical capacitated vehicles, $|K| = m$
- **$q_i$** demand of customer $i \in N$
- **Q** vehicle capacity
- **T** length of the planning horizon
Definition of the Multi-Trip VRP II

Definitions

- **A trip** is a sequence of customers, whose total demand does not exceed $Q$, that can be visited by a vehicle between two visits at the depot and that has a fixed departure time from the depot.

- **A journey** is a sequence of non-overlapping trips assigned to a vehicle whose total travel time does not exceed $T$.

The MTVRP aims at defining a set of at most $m$ journeys such that:

1. each customer is visited exactly once
2. the total traveled time is minimized
Models with 3- and 4-index Variables

4-index Variables

$x_{ij}^{kh} \in \{0, 1\}$ equal to 1 if trip $h$ of vehicle $k \in K$ traverses arc $(i,j) \in A$ (0 otherwise)

3-index Variables with Vehicle Index (without Trip Index)

$x_{ij}^{k} \in \{0, 1\}$ equal to 1 if vehicle $k \in K$ traverses arc $(i,j) \in A$ (0 otherwise)

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$x_{ij}^{h} \in \{0, 1\}$ equal to 1 if trip $h$ traverses arc $(i,j) \in A$ (0 otherwise)
## Models with 3- and 4-index Variables

### 4-index Variables

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### 3-index Variables with Vehicle Index (without Trip Index)

\[ x_{ij}^k \in \{0, 1\} \text{ equal to 1 if vehicle } k \in K \text{ traverses arc } (i, j) \in A \text{ (0 otherwise)} \]

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Models with 3- and 4-index Variables

Pros and Cons

- Polynomial number of variables
- Can be solved with commercial solvers
- Easy to embed additional side constraints

- High integrality gaps
- BigM constraints
- Symmetries in the vehicles
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2-Index Arc-based Model (Koc and Karaoglan (2011)) I

Variables

\( x_{ij} \in \{0, 1\} \) equal to 1 if arc \((i, j) \in A\) is traversed (0 otherwise)

\( x'_{ij} \in \{0, 1\} \) equal to 1 if a vehicle visits customers \(i, j \in N\) \((i \neq j)\) consecutively with a stop at the depot in between (0 otherwise)

\( \ell_i \in \mathbb{R}_+ \) load on board after visiting customer \(i \in N\)

\( a_i \in \mathbb{R}_+ \) arrival time at customer \(i \in N\)
2-Index Arc-based Model (Koc and Karaoglan (2011)) II

\[
\begin{align*}
\text{min} & \quad \sum_{(i,j) \in A} t_{ij} x_{ij} \\
\text{s.t.} & \quad \sum_{(i,j) \in A} x_{ij} = 1 \quad i \in N \quad \text{[Serve each customer]} \\
& \quad \sum_{(i,j) \in A} x_{ij} = \sum_{(j,i) \in A} x_{ji} \quad i \in V \quad \text{[Flow conservation]} \\
& \quad \ell_i + q_j \leq \ell_j + Q(1 - x_{ij}) \quad i \in N \ j \in V \quad \text{[Subtour + Load on board]} \\
& \quad a_i + t_{ij} \leq a_j + T(1 - x_{ij}) \quad i \in V \ j \in N \quad \text{[Subtour + Arrival time]} \\
& \quad a_i + (t_{i0} + t_{0j}) \leq a_j + T(1 - x'_{ij}) \quad i, j \in N : i \neq j \quad \text{[Arrival time depot visit]} \\
& \quad t_{oi} \leq a_i \leq T - t_{i0} \quad i \in N \quad \text{[Planning horizon]} \\
& \quad \sum_{j \in N} x'_{ij} \leq x_{i0} \quad i \in N \quad \text{[Link x with x']} \\
& \quad \sum_{j \in N} x'_{ij} \leq x_{0j} \quad j \in N \quad \text{[Link x with x']} \\
& \quad \sum_{(0,j) \in A} x_{0j} - \sum_{i,j \in N : i \neq j} x'_{ij} \leq m \quad \text{[Number of vehicles]} \\
& \quad x_{ij} \in \{0, 1\} \quad (i,j) \in A \\
& \quad x'_{ij} \in \{0, 1\} \quad i, j \in N : i \neq j \\
& \quad q_i \leq \ell_i \leq Q, \quad a_i \in \mathbb{R}_+ \quad i \in N \\
\end{align*}
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2-Index Arc-based Model (Koc and Karaoglan (2011))

Pros and Cons

- Polynomial number of variables (much fewer than 3- and 4-index models)
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- Instances with 50 customers are already difficult to close
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Triplet-based Model (Mingozzi, Roberti, and Toth (2013))

- $\mathcal{H}$ set of all feasible trips
- $c_h$ cost of trip $h \in \mathcal{H}$
- $\alpha_{ih}$ trip $h \in \mathcal{H}$ serves customer $i \in N$ ($\alpha_{ih} = 1$) or not ($\alpha_{ih} = 0$)
- $d_h$ duration of trip $h \in \mathcal{H}$

**Variables**

- \( x_{hk} \in \{0, 1\} \) trip $h \in \mathcal{H}$ is assigned to vehicle $k \in K$ ($x_{hk} = 1$) or not ($x_{hk} = 0$)

\[
\begin{align*}
\min & \quad \sum_{h \in \mathcal{H}} \sum_{k \in K} c_h x_{hk} & \text{[Minimize travel costs]} \quad (2a) \\
\text{s.t.} & \quad \sum_{h \in \mathcal{H}} \sum_{k \in K} \alpha_{ih} x_{hk} = 1 & i \in N \quad \text{[Serve each customer]} \quad (2b) \\
& \quad \sum_{h \in \mathcal{H}} d_h x_{hk} \leq T & k \in K \quad \text{[Planning horizon]} \quad (2c) \\
& \quad x_{hk} \in \{0, 1\} & h \in \mathcal{H} \quad k \in K \quad (2d)
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Trip-based Model (Mingozzi, Roberti, and Toth (2013))

\[ \mathcal{H} \] set of all feasible trips
\[ c_h \] cost of trip \( h \in \mathcal{H} \)
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[Minimize travel costs] (2a)

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Trip-based Model (Mingoazzi, Roberti, and Toth (2013))

Pros and Cons

- Small integrality gaps
- Instances with 100-120 customers can be closed
- Easy to embed side constraints

- Exponential number of variables
- Symmetries in the vehicles
- Column generation/branch(-and-cut)-and-price needed
- Additional constraints can make the pricing problem difficult
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Mathematical Models for the MTVRP

**Journey-based Model (Mingozzi, Roberti, and Toth (2013))**

\( \mathcal{R} \) set of all feasible journeys

\( c_r \) cost of journey \( r \in \mathcal{R} \)

\( \alpha_{ir} \) journey \( r \in \mathcal{R} \) serves customer \( i \in N \) (\( \alpha_{ir} = 1 \)) or not (\( \alpha_{ir} = 0 \))

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\text{s.t.} & \quad \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 & i \in N & \text{[Serve each customer]} \\
& \quad \sum_{r \in \mathcal{R}} x_r \leq m & & \text{[Number of vehicles]} \\
& \quad x_r \in \{0, 1\} & r \in \mathcal{R} & \text{[3d]}
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\[ R \]  set of all feasible journeys

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Variables

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\text{min} & \sum_{r \in R} c_r x_r & \text{[Minimize travel costs]} \quad (3a) \\
\text{s.t.} & \sum_{r \in R} \alpha_{ir} x_r = 1 \quad i \in N & \text{[Serve each customer]} \quad (3b) \\
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Main Side Constraints and Academic Extensions

- **Time Windows**: each customer $i \in N$ must be visited within a time interval $[a_i, b_i]$
- **Service-Dependent Loading Times**: vehicle loading time at the depot depends on the customers visited in the next trip
- **Limited Trip Duration**: maximum time between the departure from the depot and the arrival time at the last customer of the trip
- **Profits**: a profit $p_i$ is associated with each customer $i \in N$; hierarchical objective function: maximize profit first; minimize routing cost second
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## Variants of the MTVRP

### Main Side Constraints and Academic Extensions

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Trip-based Model (Hernandez et al. (2016))

- $\mathcal{H}$: set of all feasible trips
- $c_h$: cost of trip $h \in \mathcal{H}$
- $\alpha_{ih}$: trip $h \in \mathcal{H}$ serves customer $i \in N$ ($\alpha_{ih} = 1$) or not ($\alpha_{ih} = 0$)
- $\tau_{th}$: trip $h \in \mathcal{H}$ is active at time $t \in [a_0, b_0]$ ($\tau_{th} = 1$) or not ($\tau_{th} = 0$)

### Variables

- $x_h \in \{0, 1\}$: trip $h \in \mathcal{H}$ is selected ($x_h = 1$) or not ($x_h = 0$)

### Objective Function

\[
\min \sum_{h \in \mathcal{H}} c_h x_h
\]

[Minimize travel costs] (4a)

### Constraints

\[
\sum_{h \in \mathcal{H}} \alpha_{ih} x_h = 1 \quad i \in N
\]

[Serve each customer] (4b)

\[
\sum_{h \in \mathcal{H}} \tau_{th} x_h \leq m \quad t \in [a_0, b_0]
\]

[No overlaps] (4c)

\[
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(4d)
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### Formulation

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Mathematical Models for Variants of the MTVRP

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- Column generation/branch(-and-cut)-and-price needed
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- Constraints (4c) to add in a cutting plane fashion
- Instances with 25 customers can be out of reach
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- \( \mathcal{R} \): set of all feasible journeys
- \( c_r \): cost of journey \( r \in \mathcal{R} \)
- \( \alpha_{ir} \): journey \( r \in \mathcal{R} \) serves customer \( i \in N \) (\( \alpha_{ir} = 1 \)) or not (\( \alpha_{ir} = 0 \))

Variables

- \( x_r \in \{0, 1\} \): journey \( r \in \mathcal{R} \) is selected (\( x_r = 1 \)) or not (\( x_r = 0 \))

\[
\begin{align*}
\min & \quad \sum_{r \in \mathcal{R}} c_r x_r \quad \text{[Minimize travel costs]} \quad (5a) \\
\text{s.t.} & \quad \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 \quad i \in N \quad \text{[Serve each customer]} \quad (5b) \\
& \quad \sum_{r \in \mathcal{R}} x_r \leq m \quad \text{[Number of vehicles]} \quad (5c) \\
& \quad x_r \in \{0, 1\} \quad r \in \mathcal{R} \quad (5d)
\end{align*}
\]
Journey-based Model (Hernandez et al. (2014, 2016))

\( \mathcal{R} \)  set of all feasible journeys \\
\( c_r \)  cost of journey \( r \in \mathcal{R} \) \\
\( \alpha_{ir} \) journey \( r \in \mathcal{R} \) serves customer \( i \in N \) (\( \alpha_{ir} = 1 \)) or not (\( \alpha_{ir} = 0 \))

Variables

\( x_r \in \{0, 1\} \) journey \( r \in \mathcal{R} \) is selected (\( x_r = 1 \)) or not (\( x_r = 0 \))

\[
\begin{align*}
\min & \sum_{r \in \mathcal{R}} c_r x_r & & \text{[Minimize travel costs]} \\
\text{s.t.} & \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 & i \in N & \text{[Serve each customer]} \\
& \sum_{r \in \mathcal{R}} x_r \leq m & & \text{[Number of vehicles]} \\
& x_r \in \{0, 1\} & r \in \mathcal{R} & \text{(5d)}
\end{align*}
\]
Journey-based Model (Hernandez et al. (2014, 2016))

\[ \mathcal{R} \text{ set of all feasible journeys} \]
\[ c_r \text{ cost of journey } r \in \mathcal{R} \]
\[ \alpha_{ir} \text{ journey } r \in \mathcal{R} \text{ serves customer } i \in N (\alpha_{ir} = 1) \text{ or not } (\alpha_{ir} = 0) \]

Variables

\[ x_r \in \{0, 1\} \text{ journey } r \in \mathcal{R} \text{ is selected } (x_r = 1) \text{ or not } (x_r = 0) \]

\[
\begin{align*}
\min & \sum_{r \in \mathcal{R}} c_r x_r & \text{[Minimize travel costs]} \\
\text{s.t.} & \sum_{r \in \mathcal{R}} \alpha_{ir} x_r = 1 & i \in N \text{ [Serve each customer]} \\
& \sum_{r \in \mathcal{R}} x_r \leq m & \text{[Number of vehicles]} \\
& x_r \in \{0, 1\} & r \in \mathcal{R} \text{ (5d)}
\end{align*}
\]
Journey-based Model (Hernandez et al. (2014, 2016))

Pros and Cons

- Small integrality gaps (smaller than trip-based model)
- Easy to embed additional side constraints both related to trips and journeys

- Exponential number of variables
- Column generation/branch(and-cut)-and-price needed
- Pricing problem more difficult than trip-based model
- Instances with 25 customers can be out of reach
Journey-based Model (Hernandez et al. (2014, 2016))

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The Concept of Structure

Definition of Structure

A structure \( s = (0, i_1, i_2, \ldots, i_{\mu_s}, 0) \) is an ordered set of \( \mu_s \) customers that can be visited in between two visits at the depot and can start from the depot within time interval \([e_s, \ell_s]\), such that:

1. capacity constraints are satisfied
2. the duration \( d_s \) and the cost \( c_s \) are constant for each departure time from the depot within \([e_s, \ell_s]\)
3. the duration \( d_s \) is the minimum duration to serve the set of customers in the given order

\[
\begin{align*}
9:00 & \quad 9:10 & \quad 9:25 & \quad 9:38 & \quad 9:45 & \quad 9:52 \\
\text{D} & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad \text{D} \\
9:01 & \quad 9:11 & \quad 9:26 & \quad 9:39 & \quad 9:46 & \quad 9:53 \\
\text{D} & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad \text{D} \\
9:02 & \quad 9:12 & \quad 9:27 & \quad 9:40 & \quad 9:47 & \quad 9:54 \\
\text{D} & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad \text{D}
\end{align*}
\]

\[
\begin{align*}
9:00..9:02 & \quad 9:25..9:27 & \quad 9:45..9:47 \\
\text{D} & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad \text{D} \\
9:10..9:12 & \quad 9:38..9:40 & \quad 9:52..9:54
\end{align*}
\]
**Structure-based Model (Paradiso et al. (2019))**

- $\mathcal{S}$ set of all feasible structures
- $c_s$ cost of structure $s \in \mathcal{S}$
- $\alpha_{is}$ structure $s \in \mathcal{S}$ serves $i \in \mathcal{N}$ ($\alpha_{is} = 1$) or not ($\alpha_{is} = 0$)

**Variables**

- $x_s \in \{0, 1\}$ structure $s \in \mathcal{S}$ is selected ($x_s = 1$) or not ($x_s = 0$)

\[
\begin{align*}
\min & \quad \sum_{s \in \mathcal{S}} c_s x_s \quad \text{[Minimize travel costs]} \\
\text{s.t.} & \quad \sum_{s \in \mathcal{S}} \alpha_{is} x_s = 1 \quad i \in \mathcal{N} \quad \text{[Serve each customer]} \\
& \quad \sum_{s \in \mathcal{S}} x_s \leq \eta_m(\mathcal{\hat{S}}) \quad \mathcal{\hat{S}} \subseteq \mathcal{S} \quad \text{[Structure feasibility constraints]} \\
& \quad x_s \in \{0, 1\} \quad s \in \mathcal{S} \quad \text{(6d)}
\end{align*}
\]

where $\eta_m(\mathcal{\hat{S}})$ is the maximum number of structures of the set $\mathcal{\hat{S}}$ that can be simultaneously in a solution given the number of vehicles $m$
Structure-based Model (Paradiso et al. (2019))

\[ S \] set of all feasible structures

\[ c_s \] cost of structure \( s \in S \)

\[ \alpha_{is} \] structure \( s \in S \) serves \( i \in N \) (\( \alpha_{is} = 1 \)) or not (\( \alpha_{is} = 0 \))

Variables

\[ x_s \in \{0, 1\} \] structure \( s \in S \) is selected (\( x_s = 1 \)) or not (\( x_s = 0 \))

\[
\begin{align*}
\min & \quad \sum_{s \in S} c_s x_s \quad & \text{[Minimize travel costs]} \\
\text{s.t.} & \quad \sum_{s \in \hat{S}} \alpha_{is} x_s = 1 \quad i \in N \quad & \text{[Serve each customer]} \\
& \quad \sum_{s \in \hat{S}} x_s \leq \eta_m(\hat{S}) \quad \hat{S} \subseteq S \quad & \text{[Structure feasibility constraints]} \\
& \quad x_s \in \{0, 1\} \quad s \in S \quad & (6d)
\end{align*}
\]

where \( \eta_m(\hat{S}) \) is the maximum number of structures of the set \( \hat{S} \) that can be simultaneously in a solution given the number of vehicles \( m \).
Mathematical Models for Variants of the MTVRP

Structure-based Model (Paradiso et al. (2019))

\( \mathcal{S} \) set of all feasible structures
\( c_s \) cost of structure \( s \in \mathcal{S} \)
\( \alpha_{is} \) structure \( s \in \mathcal{S} \) serves \( i \in \mathcal{N} \) \( (\alpha_{is} = 1) \) or not \( (\alpha_{is} = 0) \)

Variables

\( x_s \in \{0, 1\} \) structure \( s \in \mathcal{S} \) is selected \( (x_s = 1) \) or not \( (x_s = 0) \)

\[
\begin{align*}
\min & \sum_{s \in \mathcal{S}} c_s x_s \quad \text{[Minimize travel costs]} \\
\text{s.t.} & \sum_{s \in \mathcal{S}} \alpha_{is} x_s = 1 \quad i \in \mathcal{N} \quad \text{[Serve each customer]} \\
& \sum_{s \in \hat{\mathcal{S}}} x_s \leq \eta_m(\hat{\mathcal{S}}) \quad \hat{\mathcal{S}} \subseteq \mathcal{S} \quad \text{[Structure feasibility constraints]} \\
& x_s \in \{0, 1\} \quad s \in \mathcal{S} \quad \text{(6d)}
\end{align*}
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where \( \eta_m(\hat{\mathcal{S}}) \) is the maximum number of structures of the set \( \hat{\mathcal{S}} \) that can be simultaneously in a solution given the number of vehicles \( m \)
Structure-based Model (Paradiso et al. (2019))
Pros and Cons

- Small integrality gaps
- Easy to embed additional side constraints related to trips
- Fewer variables than trip-based (and journey-based) model

- Exponential number of variables
- Column generation/branch(-and-cut)-and-price needed
- Constraints (6c) to add in a cutting plane fashion
Structure-based Model (Paradiso et al. (2019))

Pros and Cons

- Small integrality gaps
- Easy to embed additional side constraints related to trips
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- Exponential number of variables
- Column generation/branch(-and-cut)-and-price needed
- Constraints (6c) to add in a cutting plane fashion
## Trip vs Journey vs Structure (-based Models)

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## Computational Results

### MTVRP with Time Windows, Loading Times

| Group | |N| | Inst | %Gap | Opt | T<sub>tot</sub> | %Gap | Opt | T<sub>tot</sub> | %Gap | Opt | T<sub>tot</sub> |
|-------|-----|-----|-----|------|-----|------|-----|-----|------|-----|-----|------|
|       | Trip-based | Journey-based | Structure-based |
|       | Intel Core i7 2670QM | Intel Core i7 2670QM | Virtual CPU 2.59GHz |
| C    | 25  | 8   | 2.24 | 8   | 108 | 2.12 | 7   | 805 | 0.73 | 8   | 19  |
| R    | 25  | 11  | 2.41 | 11  | 646 | 1.19 | 7   | 6,925 | 0.78 | 11  | 115 |
| RC   | 25  | 8   | 5.41 | 6   | 6,671 | 2.86 | 5   | 2,963 | 1.91 | 8   | 880 |
| C    | 40  | 8   | 1.51 | 7   | 2,170 | 1.51 | 7   | 2,170 |
| R    | 40  | 11  | 0.41 | 10  | 418  | 0.41 | 10  | 418 |
| RC   | 40  | 8   | 0.83 | 8   | 872  | 0.83 | 8   | 872 |
| C    | 50  | 8   | 1.41 | 3   | 3,577 | 1.41 | 3   | 3,577 |
| R    | 50  | 11  | -    | 0   | -    | -    | -   | -    |
| RC   | 50  | 8   | 0.59 | 7   | 312  | 0.59 | 7   | 312 |
## Computational Results

### MTVRP with Time Windows, Loading Times

| Group | $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|-------|-----|------|------|-----|---------|------|-----|---------|------|-----|---------|
| C     | 25  | 8    | 2.24 | 8   | 108     | 2.12 | 7   | 805     | 0.73 | 8   | 19      |
| R     | 25  | 11   | 2.41 | 11  | 646     | 1.19 | 7   | 6,925   | 0.78 | 11  | 115     |
| RC    | 25  | 8    | 5.41 | 6   | 6,671   | 2.86 | 5   | 2,963   | 1.91 | 8   | 880     |
| C     | 40  | 8    |      |     |         | 1.51 | 7   | 2,170   |      |     |         |
| R     | 40  | 11   |      |     |         | 0.41 | 10  | 418     |      |     |         |
| RC    | 40  | 8    |      |     |         | 0.83 | 8   | 872     |      |     |         |
| C     | 50  | 8    |      |     |         | 1.41 | 3   | 3,577   |      |     |         |
| R     | 50  | 11   |      |     |         |      | 0   |         |      |     |         |
| RC    | 50  | 8    |      |     |         | 0.59 | 7   | 312     |      |     |         |
## Computational Results

### MTVRP with Time Windows, Loading Times

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R. Roberti  
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### Computational Results

**MTVRP with Time Windows, Loading Times**

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*Intel Core i7 2670QM Virtual CPU 2.59GHz*
## Computational Results

### MTVRP with Time Windows, Loading Times, Limited Trip Duration

| Group | \(|N|\) | Inst | %Gap | Opt | \(T_{\text{tot}}\) | %Gap | Opt | \(T_{\text{tot}}\) |
|-------|--------|------|------|-----|----------------|------|-----|----------------|
|       |        | Trip-based | Structure-based |
|       |        | Hernandez et al. (2014) | Intel Core 2 Duo 2.10GHz | Paradiso et al. (2019) | Virtual CPU 2.59GHz |
| C     | 25     | 1.91 | 16   | 420 | 0.38 | 16 | 14 |
| R     | 25     | 0.76 | 22   | 33  | 0.25 | 22 | 2 |
| RC    | 25     | 2.35 | 11   | 18  | 0.49 | 16 | 2 |
| C     | 40     | 1.25 | 13   | 511 | 0.48 | 16 | 151 |
| R     | 40     | 1.43 | 12   | 1,738 | 1.06 | 19 | 220 |
| RC    | 40     | -    | 0    | -   | 0.67 | 2 | 11 |
| C     | 50     | 0.22 | 16   | 62  | 0.22 | 16 | 62 |
| R     | 50     | 0.22 | 22   | 20  | 0.28 | 16 | 11 |
| RC    | 50     | 0.28 | 16   | 11  |
## Computational Results

### MTVRP with Time Windows, Loading Times, Limited Trip Duration

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### Computational Results

#### MTVRP with Time Windows, Loading Times, Limited Trip Duration

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*Hernandez et al. (2014)*

*Paradiso et al. (2019)*

*Intel Core 2 Duo 2.10GHz*

*Virtual CPU 2.59GHz*
## Computational Results

### MTVRP with Time Windows, Loading Times, Limited Trip Duration

| Group | $|N|$ | Inst | %Gap | Opt | $T_{tot}$ | %Gap | Opt | $T_{tot}$ |
|-------|-----|------|------|-----|----------|------|-----|----------|
| Trip-based | | | Hernandez et al. (2014) | Intel Core 2 Duo 2.10GHz | | | | Paradiso et al. (2019) | Virtual CPU 2.59GHz | |
|       |     |      |       |     |          |      |     |          |                      | |
| C     | 25  | 16   | 1.91  | 16  | 420      | 0.38 | 16  | 14       |
| R     | 25  | 22   | 0.76  | 22  | 33       | 0.25 | 22  | 2        |
| RC    | 25  | 16   | 2.35  | 11  | 18       | 0.49 | 16  | 2        |
|       |     |      |       |     |          |      |     |          |                      | |
| C     | 40  | 16   | 1.25  | 13  | 511      | 0.48 | 16  | 151      |
| R     | 40  | 19   | 1.43  | 12  | 1,738    | 1.06 | 19  | 220      |
| RC    | 40  | 2    | -     | 0   | -        | 0.67 | 2   | 11       |
|       |     |      |       |     |          |      |     |          |                      | |
| C     | 50  | 16   |       |     |          | 0.22 | 16  | 62       |
| R     | 50  | 22   |       |     |          | 0.22 | 22  | 20       |
| RC    | 50  | 16   |       |     |          | 0.28 | 16  | 11       |
## Computational Results

**MTVRP with Time Windows, Loading Times, Limited Trip Duration**

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Conclusions

- Increasing **interest in MTVRPs**, mainly motivated by city logistics and last-mile delivery
- **Trip-based** and **journey-based** models are effective to solve the MTVRP
- To handle **side constraints**, **structure-based** models seem the better choice, even better than set-partitioning models
Conclusions

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Open Questions

- Research on MTVRPs is scarce and 50-customer instances are already challenging, how can large instances be solved?
- Are there better models (maybe models not based on arcs, structures, trips, or journeys)?
- Can we use models not based on arcs or routes to solve other VRPs?

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References I


