Multi-Trip Vehicle Routing Problems: Variants, Formulations, and Exact Methods

Tutorial - VeRoLog 2019, Seville

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Introduction

- Most of the literature on the Vehicle Routing Problem (VRP) addresses problems where each vehicle can perform at most one trip per day
- Many contributions on VRPs where vehicles can perform multiple trips have been published in the last decade
- These problems are called Multi-Trip Vehicle Routing Problems (MTVRP)

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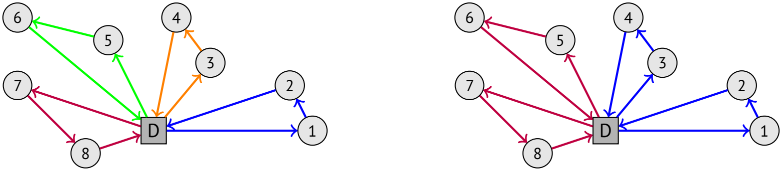

Motivation

- Such an increasing interest in MTVRPs is due, e.g., to new practices in city logistics and last-mile delivery
- The need of limiting noise and pollution in city centers requires the usage of small vans, electric vehicles, and/or drones and forbids large trucks from entering city centers
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Main Question of this Tutorial

What is the best model to solve an MTVRP to optimality?

Based on the state-of-the-art exact methods for lots of VRPs...

Set Partitioning Models!

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Definition of the Multi-Trip VRP I

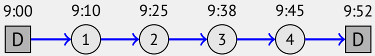
Input Data

- N set of customers
- V vertex set, $V = N \cup \{0\}$, where 0 is the depot
- $\mathcal{A}$ arc set, $\mathcal{A} = \{(i,j) \mid i,j \in V \,:\, i \neq j\}$
- $\mathcal{G}$ directed graph, $\mathcal{G} = (V, \mathcal{A})$
- t_{ij} travel time of arc $(i,j) \in \mathcal{A}$
- K fleet of identical capacitated vehicles, $|K| = m$
- q_i demand of customer $i \in N$
- Q vehicle capacity
- T length of the planning horizon

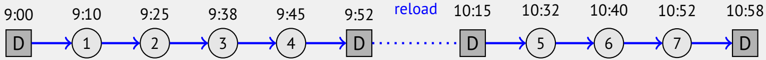
Definition of the Multi-Trip VRP II

Definitions

- A trip is a sequence of customers, whose total demand does not exceed Q, that can be visited by a vehicle between two visits at the depot and that has a fixed departure time from the depot



- A journey is a sequence of non-overlapping trips assigned to a vehicle whose total travel time does not exceed T



The MTVRP aims at defining a set of at most m journeys such that:

1. each customer is visited exactly once
2. the total traveled time is minimized

Models with 3- and 4-index Variables

4-index Variables

$x_{ij}^{kh} \in \{0,1\}$ equal to 1 if trip h of vehicle $k \in K$ traverses arc $(i,j) \in \mathcal{A}$ (0 otherwise)

3-index Variables with Vehicle Index (without Trip Index)

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Pros and Cons

- Polynomial number of variables
- Can be solved with commercial solvers
- Easy to embed additional side constraints

- High integrality gaps
- BigM constraints
- Symmetries in the vehicles

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2-Index Arc-based Model (Koc and Karaoglan (2011)) I

Variables

$x_{ij} \in \{0, 1\}$ equal to 1 if arc $(i, j) \in \mathcal{A}$ is traversed (0 otherwise)

$x'_{ij} \in \{0, 1\}$ equal to 1 if a vehicle visits customers $i, j \in N$ $(i \neq j)$ consecutively with a stop at the depot in between (0 otherwise)

$\ell_i \in \mathbb{R}_+$ load on board after visiting customer $i \in N$

$a_i \in \mathbb{R}_+$ arrival time at customer $i \in N$

2-Index Arc-based Model (Koc and Karaoglan (2011)) II

$$\min \sum_{(i,j)\in\mathcal{A}} t_{ij}x_{ij} \qquad \text{[Minimize travel times]} \qquad (1a)$$

$$\text{s.t.} \sum_{(i,j)\in\mathcal{A}} x_{ij} = 1 \qquad i \in N \qquad \text{[Serve each customer]} \qquad (1b)$$

$$\sum_{(i,j)\in\mathcal{A}} x_{ij} = \sum_{(j,i)\in\mathcal{A}} x_{ji} \qquad i \in V \qquad \text{[Flow conservation]} \qquad (1c)$$

$$\ell_i + q_j \le \ell_j + Q(1 - x_{ij}) \qquad i \in N\ j \in V \qquad \text{[Subtour + Load on board]} \qquad (1d)$$

$$a_i + t_{ij} \le a_j + T(1 - x_{ij}) \qquad i \in V\ j \in N \qquad \text{[Subtour + Arrival time]} \qquad (1e)$$

$$a_i + (t_{i0} + t_{0j}) \le a_j + T(1 - x'_{ij}) \qquad i,j \in N : i \ne j \qquad \text{[Arrival time depot visit]} \qquad (1f)$$

$$t_{0i} \le a_i \le T - t_{i0} \qquad i \in N \qquad \text{[Planning horizon]} \qquad (1g)$$

$$\sum_{j\in N} x'_{ij} \le x_{i0} \qquad i \in N \qquad \text{[Link x with x']} \qquad (1h)$$

$$\sum_{j\in N} x'_{ij} \le x_{0j} \qquad j \in N \qquad \text{[Link x with x']} \qquad (1i)$$

$$\sum_{(0,j)\in\mathcal{A}} x_{0j} - \sum_{i,j\in N : i\ne j} x'_{ij} \le m \qquad \text{[Number of vehicles]} \qquad (1j)$$

$$x_{ij} \in \{0,1\} \qquad (i,j) \in \mathcal{A} \qquad (1k)$$

$$x'_{ij} \in \{0,1\} \qquad i,j \in N : i \ne j \qquad (1l)$$

$$q_i \le \ell_i \le Q, \quad a_i \in \mathbb{R}_+ \qquad i \in N \qquad (1m)$$

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Main Side Constraints and Academic Extensions

- Time Windows: each customer $i \in N$ must be visited within a time interval $[a_i, b_i]$
- Service-Dependent Loading Times: vehicle loading time at the depot depends on the customers visited in the next trip
- Limited Trip Duration: maximum time between the departure from the depot and the arrival time at the last customer of the trip
- Profits: a profit p_i is associated with each customer $i \in N$; hierarchical objective function: maximize profit first; minimize routing cost second

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Reference	Time Windows	Service-Dependent Loading Times	Limited Trip Duration	Profits
Exact Methods				
Azi, Gendreau, and Potvin (2010)	✓	✓	✓	✓
Macedo et al. (2011)	✓	✓	✓	✓
Hernandez et al. (2014)	✓	✓	✓	
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Heuristic Methods				
Azi, Gendreau, and Potvin (2014)	✓	✓	✓	✓
Wang, Liang, and Hu (2014)	✓	✓	✓	✓
Cattaruzza, Absi, and Feillet (2016a)	✓	✓		
Anaya-Arenas et al. (2016)	✓		✓	

From Cattaruzza, Absi, and Feillet (2016b)

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$$\sum_{r \in \mathcal{R}} x_r \leq m \qquad \text{[Number of vehicles]} \tag{5c}$$

$$x_r \in \{0, 1\} \quad r \in \mathcal{R} \tag{5d}$$

Journey-based Model (Hernandez et al. (2014, 2016))

Pros and Cons

- Small integrality gaps (smaller than trip-based model)
- Easy to embed additional side constraints both related to trips and journeys

- Exponential number of variables
- Column generation/branch(and-cut)-and-price needed
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The Concept of Structure

Definition of Structure

A structure $s = (0, i_1, i_2, \ldots, i_{\mu_s}, 0)$ is an ordered set of μ_s customers that can be visited in between two visits at the depot and can start from the depot within time interval $[e_s, \ell_s]$, such that:

1. capacity constraints are satisfied
2. the duration d_s and the cost c_s are constant for each departure time from the depot within $[e_s, \ell_s]$
3. the duration d_s is the minimum duration to serve the set of customers in the given order

Structure-based Model (Paradiso et al. (2019))

$\mathcal{S}$ set of all feasible structures

c_s cost of structure $s \in \mathcal{S}$

α_{is} structure $s \in \mathcal{S}$ serves $i \in N$ ($\alpha_{is} = 1$) or not ($\alpha_{is} = 0$)

Variables

$x_s \in \{0, 1\}$ structure $s \in \mathcal{S}$ is selected ($x_s = 1$) or not ($x_s = 0$)

$$\min \sum_{s \in \mathcal{S}} c_s x_s \qquad \text{[Minimize travel costs]} \tag{6a}$$

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$$\sum_{s \in \widehat{\mathcal{S}}} x_s \leq \eta_m(\widehat{\mathcal{S}}) \qquad \widehat{\mathcal{S}} \subseteq \mathcal{S} \qquad \text{[Structure feasibility constraints]} \tag{6c}$$

$$x_s \in \{0, 1\} \qquad s \in \mathcal{S} \tag{6d}$$

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Trip vs Journey vs Structure (-based Models)

	Trip	Journey	Structure
Integrality gap	👍	👍👍	👍(👍)
Number of variables	👎👎	👎👎👎	👎
Number of constraints	👎	👍	👎👎
Trip-related constraints	👍	👍	👍
Journey-related constraints	👎	👍	👎
Complexity of algorithms	👎	👎	👎

Computational Results

MTVRP with Time Windows, Loading Times

			Trip-based Hernandez et al. (2016) Intel Core i7 2670QM			Journey-based Hernandez et al. (2016) Intel Core i7 2670QM			Structure-based Paradiso et al. (2019) Virtual CPU 2.59GHz		
Group	$\|N\|$	Inst	%Gap	Opt	T_{tot}	%Gap	Opt	T_{tot}	%Gap	Opt	T_{tot}
C	25	8	2.24	8	108	2.12	7	805	0.73	8	19
R	25	11	2.41	11	646	1.19	7	6,925	0.78	11	115
RC	25	8	5.41	6	6,671	2.86	5	2,963	1.91	8	880
C	40	8							1.51	7	2,170
R	40	11							0.41	10	418
RC	40	8							0.83	8	872
C	50	8							1.41	3	3,577
R	50	11							-	0	-
RC	50	8							0.59	7	312

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Group	$\|N\|$	Inst	%Gap	Opt	T_{tot}	%Gap	Opt	T_{tot}
C	25	16	1.91	16	420	0.38	16	14
R	25	22	0.76	22	33	0.25	22	2
RC	25	16	2.35	11	18	0.49	16	2
C	40	16	1.25	13	511	0.48	16	151
R	40	19	1.43	12	1,738	1.06	19	220
RC	40	2	-	0	-	0.67	2	11
C	50	16				0.22	16	62
R	50	22				0.22	22	20
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Conclusions

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- Trip-based and journey-based models are effective to solve the MTVRP
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Open Questions

- Research on MTVRPs is scarce and 50-customer instances are already challenging, how can large instances be solved?
- Are there better models (maybe models not based on arcs, structures, trips, or journeys)?
- Can we use models not based on arcs or routes to solve other VRPs?

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